NAG Toolbox for MATLAB

f07mj

1 Purpose

f07mj computes the inverse of a real symmetric indefinite matrix A, where A has been factorized by f07md.

2 Syntax

```
[a, info] = f07mj(uplo, a, ipiv, 'n', n)
```

3 Description

f07mj is used to compute the inverse of a real symmetric indefinite matrix A, the function must be preceded by a call to f07md, which computes the Bunch–Kaufman factorization of A.

If **uplo** = 'U', $A = PUDU^{\mathrm{T}}P^{\mathrm{T}}$ and A^{-1} is computed by solving $U^{\mathrm{T}}P^{\mathrm{T}}XPU = D^{-1}$ for X.

If **uplo** = 'L', $A = PLDL^{\mathrm{T}}P^{\mathrm{T}}$ and A^{-1} is computed by solving $L^{\mathrm{T}}P^{\mathrm{T}}XPL = D^{-1}$ for X.

4 References

Du Croz J J and Higham N J 1992 Stability of methods for matrix inversion *IMA J. Numer. Anal.* **12** 1–19

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo – string**

Indicates how A has been factorized.

uplo = 'U'

$A = PUDU^{\mathrm{T}}P^{\mathrm{T}}$, where U is upper triangular.

uplo = 'L'

$A = PLDL^{\mathrm{T}}P^{\mathrm{T}}$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **a(lda,*) – double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

Details of the factorization of A, as returned by f07md.

3: **ipiv(*) – int32 array**

Note: the dimension of the array **ipiv** must be at least $\max(1, \mathbf{n})$.

Details of the interchanges and the block structure of D, as returned by f07md.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The second dimension of the array **a** The dimension of the array **ipiv**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, work

5.4 Output Parameters

1: **a(lda,*) – double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The factorization contains the n by n symmetric matrix A^{-1}.

If **uplo** = 'U', the upper triangle of A^{-1} is stored in the upper triangular part of the array.

If **uplo** = 'L', the lower triangle of A^{-1} is stored in the lower triangular part of the array.

2: **info – int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **a**, 4: **lda**, 5: **ipiv**, 6: **work**, 7: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** $= i$, $d(i,i)$ is exactly zero; D is singular and the inverse of A cannot be computed.

7 Accuracy

The computed inverse X satisfies a bound of the form

if **uplo** = 'U', $\left|DU^{\mathrm{T}}P^{\mathrm{T}}XPU - I\right| \leq c(n)\epsilon\left(|D|\left|U^{\mathrm{T}}\right|P^{\mathrm{T}}|X|P|U| + |D|\left|D^{-1}\right|\right)$;
if **uplo** = 'L', $\left|DL^{\mathrm{T}}P^{\mathrm{T}}XPL - I\right| \leq c(n)\epsilon\left(|D|\left|L^{\mathrm{T}}\right|P^{\mathrm{T}}|X|P|L| + |D|\left|D^{-1}\right|\right)$,

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

8 Further Comments

The total number of floating-point operations is approximately $\frac{2}{3}n^3$.

The complex analogues of this function are f07mw for Hermitian matrices and f07nw for symmetric matrices.

9 Example

```
uplo = 'L';
a = [2.07, 0, 0, 0;
     3.87, -0.21, 0, 0;
     4.2, 1.87, 1.15, 0;
     -1.15, 0.63, 2.06, -1.81];
[a, ipiv, info] = f07md(uplo, a);
[aOut, info] = f07mj(uplo, a, ipiv)
```

```
aOut =
    0.7485         0         0         0
    0.5221   -0.1605         0         0
   -1.0058   -0.3131    1.3501         0
   -1.4386   -0.7440    2.0667    2.4547
info =
           0
```